

I. *The * Dissertation concerning the Figure of the Earth continued; by the Reverend J. T. Desaguliers, LL. D. F. R. S.*

HOW the Figure of the Earth is deduc'd from the Laws of Gravity and Centrifugal Force, is very well shewn by the late *Savilian* Professor of Astronomy, Dr. *John Keill*, in a Book that he wrote in the Year 1698, against Dr. *Burnet's* Theory of the Earth; and therefore I shall transcribe from him what he has said upon that Subject; because, otherwise, I shou'd only say the same Thing in other Words.

I own indeed that he has made a Mistake in that Book concerning the Measure of the Degrees of an Ellipse; but I find that all that relates to the oblate Spheroidical Figure of the Earth is right; and the little Difference of taking 15 *Paris* Feet for the Space that a Body falls thro' in a Second, instead of 15 Feet 1 Inch and 2 Lines, and a Number of Feet, a little less than true, for the Diameter of the Earth (which was not so well known at that Time) will no way invalidate his Demonstration and Proof. Here follow his Words.

“ To prove the Earth to be higher at the Equator
 “ than at the Poles, I will suppose first, that, at the Be-
 “ ginning of the World, the Earth was fluid and spheri-
 “ cal; but afterwards God Almighty having given it a
 “ Motion round its own Axis, all Bodies upon the Earth
 “ wou'd describe either the Equator or Circles, paral-
 “ lel to the Equator, and, by consequence, all wou'd
 “ endeavour to recede from the Center of their Motion.

* *V. Philos. Transact. N. 386.*

“ It is to be here observ’d, that if a Body doth freely
 “ revolve in a Circle about a Center, as the Planets do
 “ about the Sun, that its Centripetal Force (or that Force
 “ by which it is drawn towards the Center) is always
 “ equal to its Force, by which it doth endeavour to re-
 “ ceede from the Center; for the Force, which detains a
 “ Body in its Orbit, must be equal to the Force by
 “ which it endeavours to recede from its Orbit, and fly
 “ off in the Tangent. This may be clear by the Ex-
 “ ample of a Body turn’d round a Center by the Help
 “ of a Thread, which detains the Body in its Orbit;
 “ the Thread, being stretch’d by the Motion of the Body,
 “ will endeavour to contract itself equally towards both
 “ Ends, by which it will pull the Center as much to-
 “ wards the Body, as it doth the Body towards the
 “ Center.

“ Now this Centrifugal Force is always proportional
 “ to the Periphery, which each Body describes in its
 “ diurnal Motion by the first *Theorem* of *Hugenius de*
 “ *Vi Centrifuga*: So that under the *Æ*quator, which
 “ is the biggest Circle, the Centrifugal Force wou’d be
 “ greatest, and still grow less as we approach the Pole
 “ where it quite vanisheth, there being there no diurnal
 “ Rotation. And without doubt, all Bodies having this
 “ Centrifugal Force, by which they endeavour to re-
 “ ceede from the Center of their Motion, wou’d fly off
 “ from the Earth, if they were not kept in their Orbit
 “ by their Gravity, or that Force by which they are
 “ press’d towards the Center of the Earth, which is
 “ much stronger upon our Earth than the Centrifugal
 “ Force; and because the Gravity upon the Surface of
 “ the Earth is always the same; but the Centrifugal
 “ Force alters and grows less, the nearer we come to
 “ the Poles, it is plain that the Gravity under the *Æ*-
 “ quator, having a greater Force to oppose it, than that
 which

“ which is near the Poles, will not act so strongly in
 “ the one Place as in the other, and consequently Bo-
 “ dies will not be so heavy under the *Æquator* as at
 “ the Poles. ——— If the Circle * *Æ P Q P* represent
 “ the Earth, *Æ Q* the *Æquator*, and *P P* the Poles, if
 “ *C* be a Body in the *Æquator*, it is evident that it will
 “ be pull’d by two contrary Forces; namely, that of
 “ its Gravity, which pulls it towards the Center, and
 “ that of its Centrifugal Force, which pulls it from it.
 “ Now, if both these Forces were equal, it is evident
 “ it wou’d go neither of these Ways; but if one were
 “ stronger than the other, it wou’d move where the
 “ strongest Force pulls it, but only with a Velocity
 “ which is proportional to the Differences of these two
 “ Forces, and therefore it wou’d not descend so fast as
 “ if there were no Centrifugal Force, pulling against it;
 “ that is, a Body in the *Æquator*, does press less towards
 “ the Center, than at the Pole, where there is no Cen-
 “ trifugal Force to lessen its Gravity. Bodies therefore,
 “ of the same Density, are not so heavy in one Place as
 “ in the other.

“ Now in a spherical Fluid, all whose Parts gravitate
 “ towards the Center, I think it is evident from the
 “ Principles of Hydrostaticks and Fluidity, that all those
 “ Bodies, which are equally distant from the Center,
 “ must be equally press’d with the Weight of the incum-
 “ bent Fluid, and if one Part come to be more press’d
 “ than another, that which is most press’d will thrust
 “ that out of its Place which is least, till all the Parts
 “ come to an *Æquilibrium* one with another; and this
 “ is known by a common and easy Experiment, if you
 “ take a recurv’d Tube, † and fill it with Water or any
 “ other Fluid, it will rite equally in both Legs of the

* Fig. I.

† Fig II.

“ Tube, so that the Surfaces CE and FI are equally
 “ press’d by the incumbent Columns BCE , and
 “ $GFIH$, but if one of the Legs of this Tube shou’d
 “ be fill’d with Oil, or some other lighter Fluid, and
 “ the other with Water, the lighter Fluid will rise
 “ higher than the other, for otherwise, these Surfaces,
 “ which are equally distant from the Center, wou’d not
 “ be equally press’d.

“ Just so if $P\text{Æ}MPS$, * represents a fluid Sphere,
 “ which we may imagine compos’d of a great many
 “ communicating Canals or Tubes, the Fluid in every
 “ one of which presses upon the Center; now if the
 “ Fluid, in every one of these Tubes, was of equal
 “ Weight or Gravity, it is plain, that, by that means,
 “ they wou’d be also of an equal Height from the Cen-
 “ ter; for by that means only, wou’d the Center be e-
 “ qually press’d by the Weight of all the Tubes; but
 “ if the Fluid, in the Canal $\text{Æ}OM$, were lighter than
 “ the Fluid in the Canal POS , it is plain, that in this
 “ Case, the Fluid POS , pressing more on the Center,
 “ than the Fluid in the Canal $\text{Æ}OM$, the Surface of
 “ the Fluid $\text{Æ}OM$, will rise to a greater Height or
 “ Distance from the Center; so that by its greater
 “ Height, which recompenses its lesser Gravitation, it
 “ will press equally upon the Center with the Fluid in
 “ the Canal POS . After the same manner, † if the
 “ Fluid in the Canal GOH , were heavier than the
 “ Fluid in the Canal $\text{Æ}OM$, but lighter than that
 “ which is in POS , then wou’d the Canal GOH be
 “ shorter than $\text{Æ}OM$, but longer than POS , and the
 “ Figure compos’d of all these Tubes, wou’d be in the
 “ Form of a Spheroid which is generated by the Cir-
 “ cumrotation of a Semi-ellipsis round its Axis; but as I

“ have already shew’d, that if ÆOM represent the Se-
 “ midiameter of the Æquator , that all Bodies in it are
 “ lighter than in POS , the Axis of the Æquator
 “ (we take the Diameter and Axis here, not as pure
 “ Mathematical Lines, but as small Canals or Tubes.)
 “ and just so those Bodies which are in the Tube GOH ,
 “ I have prov’d to be lighter than those in POS , but
 “ but heavier than the Bodies which are in ÆOM ,
 “ the Centrifugal Force in GH being less than that
 “ which is in ÆM , and there is no Centrifugal Force
 “ in the Poles PS . It is plain, therefore, that the Tube
 “ ÆOM will be longer than GOH , and GOH will
 “ be longer than POS , that is, the Diameter of the
 “ Æquator , will be longer than the Axis of the Earth,
 “ and consequently the Figure of the Earth will be af-
 “ ter the Fashion of a broad Spheroid, which is gene-
 “ rated by the Rotation of a Semi-ellipsis round its les-
 “ ser Axis. This, I hope, will be sufficient to convince
 “ *Theorist* of the Falseness of his own Assertion, since
 “ it is plain Demonstration, than an Earth, form’d from
 “ a Chaos, must have a very different Figure from what
 “ he supposes it had.

“ But I will now proceed farther, and inquire how
 “ much the Gravity is diminish’d at the Æquator , or
 “ any other Parallel by the Centrifugal Force, which
 “ all Bodies acquire by being turn’d round the Earth’s
 “ Axis, that from thence we may endeavour to deter-
 “ mine, what Proportion the Diameter of the Earth’s
 “ Æquator has to its Axis; to calculate which, I will
 “ first suppose, that the mean Semidiameter of the
 “ Earth is 19615800 *Paris* Feet, according to the late
 “ Observations of the *French* Mathematicians, and since
 “ the Earth turns round its Axis in the Space of 23
 “ Hours, 56’, for in that Time, the same Meridian re-
 “ turns to the same immoveable Point of the Heaven
 “ again

“ again (but the Sun, in the mean time, seeming to be
 “ mov’d a Degree, according to the Series of the Signs,
 “ is the Cause why there are four Minutes more requir’d
 “ before the Meridian can overtake him) from thence
 “ it follows, that a Body, under the Æquator, moves
 “ through 1426,88 Feet, in the Space of one Second of
 “ Time. Now, according to the Theorem given us by
 “ Sir *Isaac Newton* in his *Philosophiæ Naturalis*
 “ *Principia Mathematica*, Schol. Prop. 4. Lib 1. The
 “ Centrifugal Force of any Body has the same Propor-
 “ tion to the Force of Gravity, that the Square of the
 “ Arch, which a Body describes in a given Time, di-
 “ vided by its Diameter, has to the Space, through which
 “ a heavy Body moves, in falling from a Place in which
 “ it was at rest in the same Time; and supposing a heavy
 “ Body falls 15 Foot in a Second of Time, by Calcula-
 “ tion, it will from thence follow, that the Force of
 “ Gravity has the same Proportion to the Centrifugal
 “ Force at the Æquator, that 289 has to Unity; and
 “ therefore by this Centrifugal Force which arises from
 “ the Diurnal Rotation of the Earth round its Axis;
 “ any Body, placed in the Æquator, loses $\frac{2}{289}$ Part of its
 “ Gravity, which it wou’d have were the Earth at rest,
 “ or which is the same Thing, a heavy Body plac’d at
 “ either of the Poles (where there is no Diurnal Rota-
 “ tion, and consequently no Centrifugal Force) which
 “ weighs 289 Pounds, if it were brought to the Æquator,
 “ wou’d weigh only 288 Pounds.

“ Having thus determin’d the Proportion of the Cen-
 “ trifugal Force, at the Æquator, to the Force of Gravity,
 “ it will be easy from thence to shew their Proportions
 “ in any Parallel; for it is compounded of the Pro-
 “ portion of One to 289, and of the Co-sine of the La-
 “ titude to the Radius; for if two Bodies describe diffe-
 “ rent Peripheries in the same Time, their Centrifugal
 “ Forces

“ Forces are proportional to their Peripheries, or to the
 “ Semi-diameters of these Peripheries, as is determin’d
 “ by Monf. *Hugens*, in his *Theoremata de Vi Centri-*
 “ *fuga & Motu circulari*; but the Periphery which
 “ a Body in the Æquator describes, has its Semi-dia-
 “ meter equal to the Radius or Semi-diameter of the Earth,
 “ and in any other Place, the Parallels, in which Bodies
 “ move, have the Co-sines of their Latitude for their
 “ Semi-diameters, and therefore it will follow, that the
 “ Force of Gravity is to the Centrifugal Force in a Pro-
 “ portion, compounded of the Radius to the Co-sine of
 “ the Latitude, and of 289 to 1. and therefore at the
 “ Latitude of $51^{\circ} 46'$ (for Example) it will be as
 “ 466 to 1.

“ But we must observe, that it does not from thence
 “ follow, that a Body in that Latitude loses $\frac{1}{478}$ Part of
 “ its absolute Gravity, which it wou’d have, were the
 “ Earth at rest. For that cou’d not be, unless the Cen-
 “ trifugal Force acted directly contrary to the Force of
 “ Gravity, which it doth no where but at the Æquator;
 “ for let the Circle * Q P E represent the Earth, Q E
 “ the Diameter of the Æquator, O its Center, and let
 “ B represent a Body which we suppose to hang by the
 “ Thread A B, and to be placed any where between the
 “ Pole P and the Æquator Q, and let B D be drawn per-
 “ pendicular to the Axis. It is plain, that if the Earth had
 “ had no Diurnal Rotation, the Body B wou’d draw the
 “ Thread A B into the Position of A C, since by that
 “ means it descends as near as it can to the Center, and
 “ there it wou’d stretch the Thread with all the Force
 “ of its Gravity; or if we will suppose, that the Centri-
 “ fugal Force acted according to the same Direction A C,
 “ it wou’d then directly oppose the Force of Gravity,

* Fig. V.

“ and the Thread wou'd remain in the same Position,
 “ but it wou'd be stretch'd with a Force proportional
 “ to the Differences of these two Forces.

“ But because the Body B turns round the Center D,
 “ it will endeavour to recede from it according to the
 “ Line C B, in which Direction the Centrifugal Force
 “ acting, it will not directly oppose the Force of Gra-
 “ vity, but it will draw the Thread from the Position
 “ A C into the Position A B, let B G be drawn perpen-
 “ dicular to A C; if B C represent the Centrifugal
 “ Force, acting according to the Direction B C, it is equi-
 “ valent (as is commonly known) to two Forces, one
 “ of which is as G C, and acts according to the Dire-
 “ ction C G, which is contrary to that by which it de-
 “ scends to O, the other is as G B, and acts according
 “ to the Direction G B, which is no way contrary to
 “ the Force of Gravity. If therefore B C represent the
 “ total Centrifugal Force of the Body B, that Part of it,
 “ which directly opposes the Force of Gravity, will be
 “ G C; from whence it follows, that the Decrease of
 “ Gravity, in going from the Pole to the Æquator, is al-
 “ ways as the Square of the Co-sine of the Latitude;
 “ for draw B H parallel to the Axis P P, and because
 “ the Triangles H C B, C D O are Equi-angular, there-
 “ fore H C is to C B as C O is to C D, or as Q O is to
 “ C D, but Q O is to C D as the Decrease of Gravity
 “ at Q is to the Centrifugal Force at C. And there-
 “ fore H C is to C B, as the Decrease of Gravity at Q,
 “ is to the Centrifugal Force at C. But if C B repre-
 “ sent the Centrifugal Force at C, G C will represent
 “ that Part of it which acts directly against the Force
 “ of Gravity, and consequently the Decrease of Gra-
 “ vity at the Æquator is to the Decrease of Gravity at C,
 “ as H C is to G C; now H C is to G C, in duplicate
 “ Proportion of H C to C B, or of C O or O Q to C D
 “ by

“ by the 8th of the 6th of *Euclid*, and therefore the
 “ Decrease of Gravity at Q is to the Decrease of Gra-
 “ vity at C, as the Square of C O is to the Square of
 “ C D, which was to be demonstrated.

“ From whence, it is plain, that if H C represent the
 “ Decrease of Gravity at the Æquator, and G C its De-
 “ crease at C, then will G H represent the Difference
 “ of these two Diminutions, or the Difference between
 “ the Gravity at Q, and the Gravity at C, but H C is
 “ to H G in duplicate Proportion of H C to H B, or
 “ of O C to D O; that is, the Decrease of Gravity at
 “ the Æquator is to its encrease at C, as the Square of the
 “ Radius is to the Square of the Sine of the Lati-
 “ tude.

“ By this also it will appear, that the Direction of
 “ heavy Bodies is not to the Center of the Earth, as has
 “ been always supposed; for if we take a heavy Body
 “ and hang it by a Thread, the Thread produced will
 “ not pass through the Center any where but at the
 “ Poles and the Æquator, for in the Figure the Thread
 “ is carry'd by the Centrifugal Force of the Body B,
 “ from the Position A C into the Position A B, where
 “ it will rest.

“ Now to determine the Angle C A B, which the
 “ Line of Direction of the Body makes with the Line
 “ A C, let A N be drawn parallel to B C, and pro-
 “ duce O B till it meet with it in N, and let us consider
 “ the Body B as drawn by three Powers, according to
 “ three different Directions B O, B L, and A B, the
 “ Power which pulls it, according to B O, is its Gravity,
 “ that which draws it, according to the Direction B L,
 “ is its Centrifugal Force, and that which acts accord-
 “ ing to A B, is the Strength of the Thread, by which
 “ the Body is hinder'd to move according to either of the
 “ two other Directions, and therefore it is an *Æquili-*
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“ *brium* with the other two Powers; but by a Theorem
 “ which is demonſtrated by ſeveral of the Writers of
 “ Mechanics, but particularly by Monſ. *Huygens* in his
 “ ſmall Treatiſe *De Potentiis per Fila trahentibus*.
 “ If a Body be pull’d by three different Powers which
 “ are in *Æquilibrio* with one another, according to
 “ three different Directions, *AB*, *BL* and *BO*, theſe
 “ three Powers will be as the three Sides of the Tri-
 “ angle *ABN*, *viz.* as *AB*, *AN* and *BN* reſpectively;
 “ or as *AB*, *BC* and *AC*; *BN* being very near pa-
 “ rallel, and conſequently equal to *AC*, ſince they do
 “ not meet but at a great Diſtance. From hence it fol-
 “ lows, that the Force of Gravity is to the Centrifugal
 “ Force, as *AC* to *BC*. But a Method has been al-
 “ ready ſhown, how the Proportion of the Force of
 “ Gravity to the Centrifugal Force may be determined,
 “ and therefore the Proportion of *AC* to *BC* may be
 “ be alſo determined, which at the Latitude of $51^{\circ} 46'$
 “ is as 446 to 1. Therefore in the Triangle *ABC*, the
 “ Proportion of *AC* to *BC* is known, and the Angle
 “ *ACB* being equal to the Angle *COQ*, which is ſub-
 “ tended by the Arch *CQ*, the Latitude of the Place,
 “ from thence by the Tables of Sines and Tangents,
 “ the Angle *BAC* may be known, which in the above-
 “ mentioned Latitude is about 5 Minutes.

“ From hence alſo it will appear, that it is not the
 “ Line *AC*, which being produced paſſes through the
 “ Center, but the Line *AB* that is perpendicular to the
 “ Curve *PQ*, for all the Particles of the Fluid will ſettle
 “ themſelves in ſuch a Poſition, that their Lines of Di-
 “ rection downwards, muſt be perpendicular to the Sur-
 “ face of the Body which they compoſe, for otherwiſe the
 “ Parts of the Fluid wou’d not be in *Æquilibrio* one
 “ with another, and therefore altho’ the Lines of Di-
 “ rection of heavy Bodies do not paſs through the
 “ Center

“ Center of the Earth, yet are they still perpendicular
 “ to their Horizons; and, upon this Account, there
 “ cou’d arise no Error in levelling of Lines, and in find-
 “ ing the Risings and Fallings of the Ground.

“ Upon this account also it will appear, that the
 “ Surface of the Earth is not spherical, for if it were,
 “ then wou’d all Lines, drawn from the Center, be per-
 “ pendicular to the Surface of the Earth, since it is the
 “ known Property of a Sphere that they must be so;
 “ but I have already shew’d, that it is not so in the
 “ Earth, and therefore it is plain, that the Earth is not
 “ a Sphere. That therefore I may inquire more parti-
 “ cularly into the Figure of the Earth, I will resume
 “ my former Hypothesis, that the Earth is composed of
 “ an infinite Number of Canals, which communicate
 “ with one another at the Center, and are all equipon-
 “ derant, of which we will consider two, as OQ and
 “ OC , and let $OQ = r$, $OD = x$ and $DC = y$, let
 “ the absolute Gravity be call’d p , and the Centrifugal
 “ Force at the Æquator n . OC is equal to $\sqrt{x^2 + y^2}$
 “ the Weight of the Canal OQ is equal to the absolute
 “ Gravity of the whole Canal *minus* the Centrifugal
 “ Force of each Particle contain’d in it, and because the
 “ Centrifugal Force of each Particle is as its Distance
 “ from the Center, and therefore it encreases in an A-
 “ rithmetical Progression, the greatest of which is n ,
 “ consequently the Sum of all the Centrifugal Force is
 “ equal to $\frac{1}{2} n r$, but upon the Hypothesis, that Gravity
 “ is the same at all Distances from the Center, the ab-
 “ solute Gravity of the Canal OQ is $p r$, and therefore
 “ its real Weight upon the Center OQ is $p r - \frac{1}{2} n r$,
 “ after the same Manner, the absolute Gravity of the
 “ Canal OC is $p \times \sqrt{x^2 + y^2}$; but the Sum of all
 “ the Centrifugal Forces of all the Fluids in the Canal
 “ OC , is equal to the Centrifugal Force of the Fluid in
 N n 2 “ CD

“ C D (as may be easily prov'd from the Consideration
 “ of inclin'd Planes) but the Centrifugal Force at C,
 “ being to the Centrifugal Force at Q, as C D is to O Q
 “ (that is, as y is to r) the Centrifugal Force at C will
 “ be equal to $\frac{n y}{r}$, and because the Centrifugal Force of
 “ each Particle is as its Distance from the Point D,
 “ which is the Center of the Circle that the Fluid in
 “ the Canal C D describes, and therefore the Centrifugal
 “ Forces, in counting from the Point D, must encrease
 “ in an Arithmetical Progression, the greatest of which
 “ is $\frac{n y}{r}$, and therefore the Sum of all the Centrifugal
 “ Forces in C D must be equal to $\frac{n y y}{2 r}$, therefore the
 “ Weight of the Canal O C is $= p \sqrt{x^2 + y^2} - \frac{1}{2}$
 “ $\frac{n y y}{r} = p r - \frac{1}{2} n r$, which Equation expresses the
 “ Nature of the Curve that is made by the Section of
 “ the Earth with a Plane through its Poles, and by this
 “ the Proportion of the Axis of the Earth, to the Dia-
 “ meter of the Æquator, may be easily determin'd; for
 “ when C O coincides with O P, then C D or y be-
 “ comes equal to nothing, and the Equation is $p \sqrt{x^2}$
 “ $= p r - \frac{1}{2} n r$ or $p x = p r - \frac{1}{2} n r$, and therefore
 “ by the 16th of the 6th, p has the same Proportion to
 “ $p - \frac{1}{2} n$ that r has to x , or O Q to O D, but p is to
 “ $p - \frac{1}{2} n$ as 289 is to 288 $\frac{1}{2}$, or as 578 is to 577, which
 “ therefore is the Proportion of the greatest Diameter of
 “ the Earth to the least; but this is upon Supposition,
 “ that Gravity is the same at all Distances from the
 “ Center; but if we will suppose, that the Gravity of
 “ Bodies without the Earth is in a Proportion reciprocal
 “ to the Squares of their Distances from the Center, the
 “ Gravity

“ Gravity of those Bodies, which are within the Earth,
 “ will be directly as their Distance, both which do best
 “ agree with the observ’d Phænomena of Nature ; then
 “ will the Gravity at the Æquator be to the Gravity
 “ at the Poles as 689 to 692, which Numbers, in this
 “ Hypothesis, do also express the Proportion of the Dia-
 “ meter of the Earth, drawn through its Poles, to its
 “ Diameter drawn in the Plane of the Æquator.

“ It is upon the Account of this Diminution of Gra-
 “ vity, according as we approach the Æquator, that
 “ Pendulums of the same Lengths in different Latitudes
 “ take different Times to perform their Vibrations ;
 “ for because the accelerating Force of Gravity is less
 “ at the Æquator than under any Parallel, and under
 “ any Parallel it is still less than under another which
 “ is nearer the Poles ; it does plainly from thence fol-
 “ low, that a Body plac’d in the Æquator, or in any
 “ other Parallel, will take a longer Time to descend thro’
 “ an Arch of a given Circle, than it wou’d do at the
 “ Poles, and the farther a Body is remov’d from the
 “ Poles, the longer Time it will take to descend through
 “ any given Space.

“ From hence it follows, that the Length of Pendu-
 “ lums, which perform their Vibrations in equal Times
 “ in different Latitudes, are directly as the accelerating
 “ Forces of their Gravities ; for the Time a Body takes
 “ to descend through an Arch of a Cycloid, is to the
 “ Time it will take to fall through the Axis of the Cy-
 “ cloid always in a given Proportion, *viz.* as the Semi-
 “ periphery of a Circle is to its Diameter by the 25th
 “ Prop. of *Huygen’s Horologium Oscillatorium* ; and
 “ therefore when the Times in which a Body descends
 “ through the Axes of two different Cycloids are equal,
 “ the Times of the Descent through the Cycloids will
 “ be also equal ; but when the Times of the Descent
 “ through

“ through the Axes are unequal, these Axes, and consequently the Lengths of the Pendulum which vibrates in these Cycloids, are proportional to the accelerating Forces of their Gravities.

“ By this if we know the Length of a Pendulum which performs its Vibrations in a given Time, in any one Part of the Earth, it is easy to determine the Length of a Pendulum, which performs its Vibrations in the same Time in any other Part of the Earth; as for Example, the Length of a Pendulum, which vibrates Seconds at *Paris*, is three Foot eight Lines and a Half, let it be requir'd to find the Length of a Pendulum, which vibrates Seconds at the *Æquator*. Because the Gravity at the Poles is to the Gravity at the *Æquator*, as 692 is to 689; therefore the Decrease of Gravity at the *Æquator* is $\frac{1}{3}$ Parts of the whole Gravity; but, as I have before demonstrated, the Decrease of Gravity at the *Æquator* is to its Encrease in any other Latitude, as the Square of the Radius is to the Square of the Sine of the Latitude, now the Latitude of *Paris* being $48^{\circ} 45'$, its Sine is 75.183, and therefore the Square of the Radius is to the Square of the Sine of the Latitude as 1000000 to 565248, but as 1000000 is to 565248, so is 3.000 the Number which represents the Decrease of Gravity at the *Æquator* to 1.695, the Number which represents its Encrease at *Paris*, which added to 689 the Gravity at the *Æquator*, makes 690.695, the Number which will represent the Gravity at *Paris*. But I have already shew'd, that as the Gravity at *Paris* is to the Gravity at the *Æquator*, so is the Length of a Pendulum which vibrates Seconds at *Paris*, to the Length of a Pendulum that vibrates Seconds at the *Æquator*, that is as 690, 695 to 689, so is 36,708 the Length of a Pendulum at *Paris*, which performs its

“ Vibra-

“ Vibration in a Second to 36,616, which therefore is
 “ the Length of a Pendulum which performs its Vibra-
 “ tions in a Second at the *Æquator* ; so that the Diffe-
 “ rence between these two Pendulums is $\frac{2}{3000}$ Parts of an
 “ Inch, which comes pretty near the Observations of
 “ *Monf. Richer*, who at the Island of *Cayenne*, whose
 “ Latitude is $5^{\circ} 00'$ found that a Pendulum, which vi-
 “ brates Seconds there, was a tenth Part of an Inch
 “ shorter than a Pendulum, which vibrates Seconds at
 “ *Paris*.

“ Thus we see that the Principles and Hypothesis,
 “ and withal their Consequences, upon which the broad
 “ spheroidical Figure of the Earth is founded, do exactly
 “ agree with Observations, and therefore there is no
 “ Doubt to be made, but that the Earth is really of such
 “ a Figure, and that the Hypothesis upon which this
 “ Figure is grounded (*viz.* the diurnal Rotation of the
 “ Earth, and by consequence the Centrifugal Force of
 “ all Bodies upon it) must be admitted for a true one ;
 “ since the different Vibrations of Pendulums of the
 “ same Length, in different Latitudes, can depend upon
 “ no other Cause ; for the Change of Air is not able to
 “ produce any such Effect, for if the Air made really
 “ any Alterations in the Vibrations of a Pendulum, it
 “ wou'd produce a quite contrary Effect than what is
 “ observ'd ; for Pendulums near the *Æquator* wou'd
 “ move faster than they wou'd do in Places of greater
 “ Latitude, the Air in the one Place, being more rari-
 “ fied, is much thinner and finer than it is in the other,
 “ and therefore gives less Resistance to Bodies that move
 “ in it.

“ In this Reasoning, we have suppos'd the Earth to
 “ have been at first fluid, as the *Theorist* has done be-
 “ fore us, but if we will put the Case, that the Earth
 “ was first partly fluid and partly dry, as it is at present,

“ yet

“ yet because we find that the Land is very near of the
 “ same Figure with the Sea (only rais’d a little higher
 “ that it might not be overflow’d) composing with it
 “ the same Solid, and I have already shew’d that the
 “ Surface of the Ocean is spheroidical and not spherical,
 “ there is no doubt to be made, but that the Land was
 “ form’d into the same Figure by its wise Creator at the
 “ Beginning of the World, for if it were otherwise, then
 “ wou’d the Land towards the Æquator have been over-
 “ flow’d with Water, which, as I have already prov’d,
 “ must have been higher at the Æquator than at the
 “ Poles; and therefore the Sea wou’d rise there and
 “ spread itself like an Inundation upon all the Land.”

To make an End of this long Dissertation, let us in a few Words compare the Experiments and Observations made use of to confirm each of the Opinions above-mentioned.

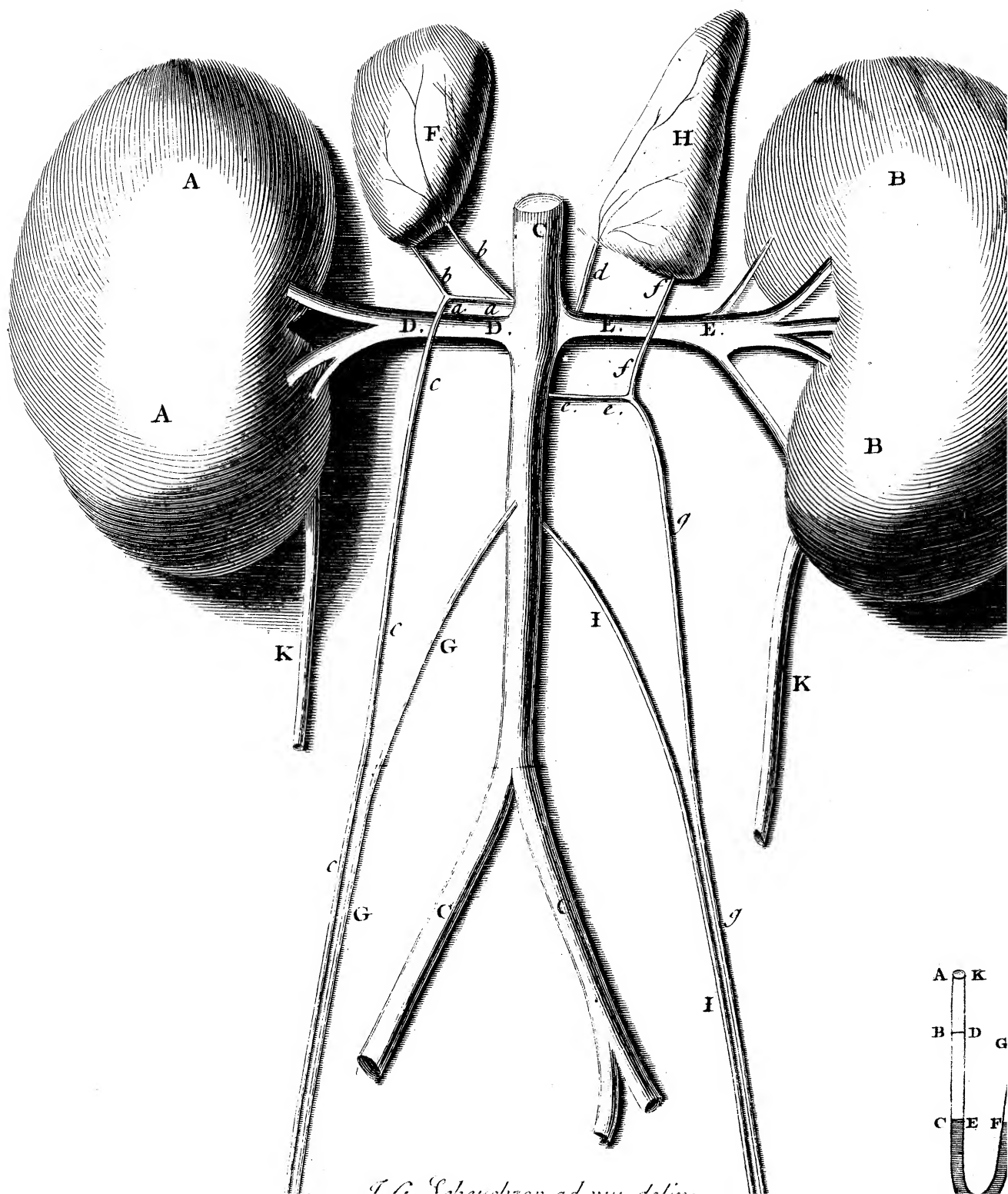
To prove Monsf. Cassini’s Figure of the Earth, we must take the Altitude of a Star nearer than to 2 Seconds; because 2 Seconds answer to 32 Toises on the Surface of the Earth, and the Difference of the Length of Degrees is but 31. And what is more, we must take this Angle with an Instrument of 39 Inches Radius; because the 10 Foot Sector was only us’d at the Ends of the two Parts of the Meridian.

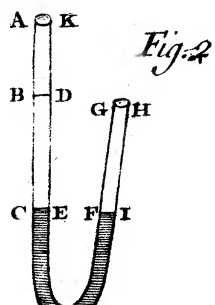
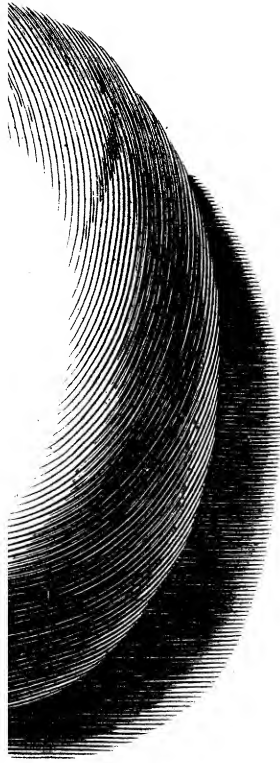
To disprove Monsf. Cassini’s Hypothesis, we need only observe whether a Plumb-Line makes an Angle of 5 Minutes with a Perpendicular to the Surface of stagnant Waters, or Lines of Level.

To prove Monsf. Cassini’s Opinion, the Height of a great many Mountains must be accurately measur’d by Trigonometry, which Mathematicians have always found very difficult.

To prove Sir Isaac Newton's Opinion, we are only to measure about one Tenth of an Inch in a Rod of 39,129 Inches; and to know what to allow for the lengthening of the same Rod by the Summer Heat, when it is shut up in a Case, and carried towards the Æquator. For though the Experiments on Pendulums, made by several Persons that travell'd Southward, differ among themselves, yet they all agree in this, that the Observers were oblig'd to shorten their Pendulums, in order to make them swing Seconds, as they went towards the Æquator. And when we come to compare them together, in order to have the exact Proportion of Length in different Latitudes; we must rely on the most exact Experimenter, which we may very well do on *Monf. Richer*: because when he found a Difference, he was so careful to find out how much it was, that he caus'd a simple Pendulum to swing, and compar'd it with a good Pendulum Clock, which he did several Times every Week for 10 Months together; and when he return'd to *France*, he compar'd it with the Length of the Pendulum at *Paris*; which is of 3 Feet 8 $\frac{1}{5}$ Lines (or 39,129 *English* Inches) and found it to be shorter by 1 $\frac{1}{4}$ Line.

Fig. 6.





J. G. Scheuchzer ad viv. delin.



Fig 1.

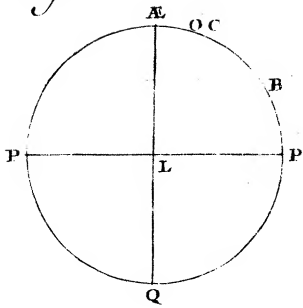


Fig 3.

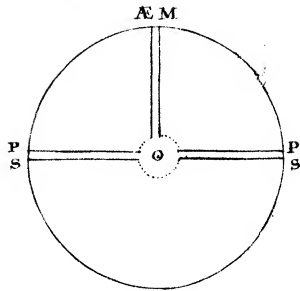
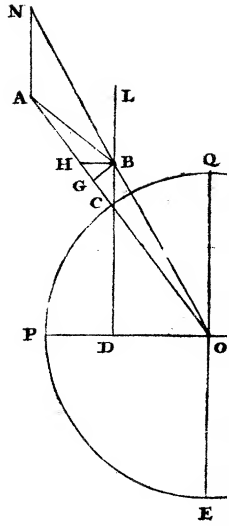
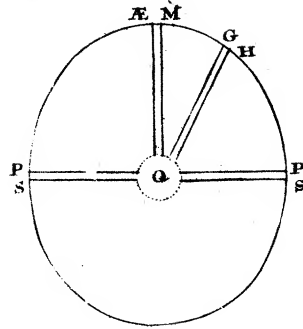
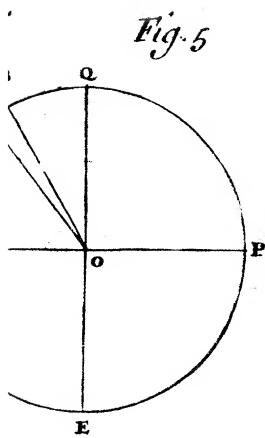


Fig 4.





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Fig 6

